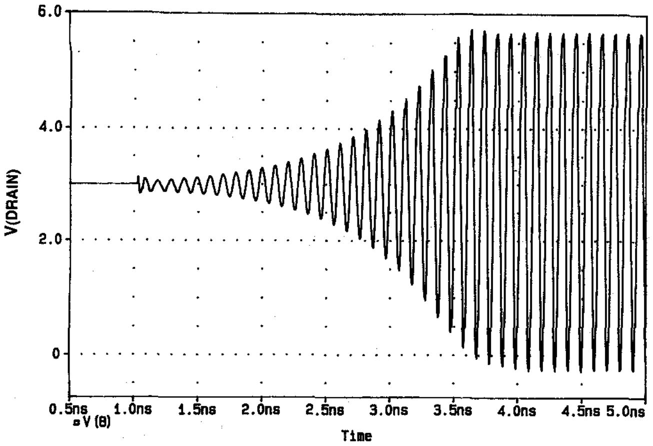

Fig. 2. Build-up of oscillation from noise level.

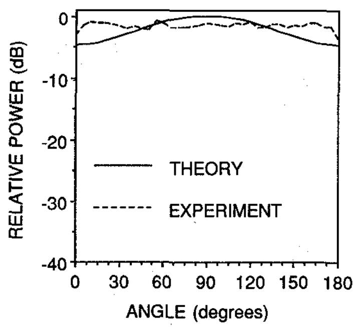

Fig. 3. Comparison of theoretical and experimental E-plane pattern.

analysis indicates that at high values of line characteristic impedances, the oscillators no longer have sufficient interaction to maintain phase locking. It was also observed that at low values of line characteristic impedances phase locking cannot be obtained and other modes are excited at different frequencies.

III. Experimental Results

As a first step a "cell" consisting of only one active device was designed and fabricated on a Duroid™ substrate of relative dielectric constant of 2.33 and thickness of 0.031 inches. The active device used was a GaAs MESFET Avantek ATF 26884, biased at $V_{DS} = 3.0$ V and $I_{DS} = 10$ mA. An effective isotropic radiated power of 36.3 mW at a frequency of 9.97 GHz was obtained. The power produced by a single device was calculated to be 7.24 mW after correcting for the antenna directivity (6.92 dB). Next, the four device combiner was fabricated and all the devices were biased at the same point. An EIRP of 484 mW was achieved at a frequency of 10.02 GHz. No other modes of oscillation were observed.

After correcting for the array directivity (12.8 dB), the power generated by each device in a four-device combiner was 6.35 mW. Hence, the power combining efficiency was 87.7%. Figs. 3 and 4 show the comparison of theoretical and experimental E-plane and H-plane patterns. A close match between the theoretical and experimental patterns was observed.

IV. Conclusion

A four-MESFET planar periodic spatial power combiner was designed and fabricated. An EIRP of 484 mW was obtained at a frequency close to the design frequency of 10.02 GHz. A power combining efficiency of 87.7% was achieved with no other modes of oscillation. A large-signal analysis of the structure was performed to study the phase locking sensitivity of the combiner. These types of structures have applications in motion detection, communication and medical applications where radiating structures are desirable.

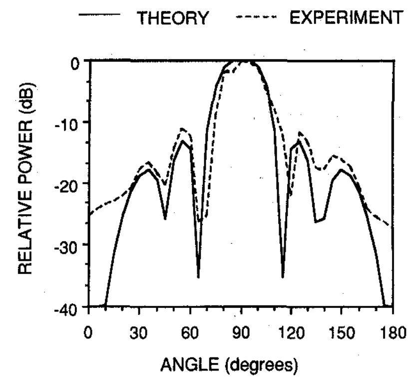

Fig. 4. Comparison of theoretical and experimental H-plane pattern.

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Proviso on the Unconditional Stability Criteria for Linear Twoport

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***Abstract*— The proviso imposed by Rollett [1] on the well-known stability criteria for linear twoports is examined and redefined as the requirement that at least one set of immittance parameters must have no RHP (right-half plane) poles. It is shown that the proviso can be interpreted as the extreme cases of a newly introduced proviso that requires that the S-parameters defined for at least one pair of arbitrary positive reference impedances have no RHP poles. The new proviso means that the twoport must be stable for at least one pair of arbitrary positive resistance terminations. Since S-parameters are much easier to measure than immittance parameters at microwaves and their direct measurability is an indication of the absence of RHP poles, the new proviso allows us to apply the stability criteria to measured circuits less consciously of the proviso.**

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I. INTRODUCTION

A linear twoport is said to be unconditionally stable if, with arbitrary passive terminations (including reactive teminations), the circuit does not oscillate. According to Rollett [1], an equivalent statement is that the real part of the immittance looking into one of the two ports remains positive with arbitrary passive termnations at the other, under the *proviso* (hereafter called the Rollett's proviso), that the characteristic frequencies of the twoport with ideal terminations (infinite immittances, i.e., open or short circuits, as appropriate) lie in the LHP (left-half plane). Under such a proviso he reformulated the necessary and sufficient conditions for unconditional stability by

$$K = \{2\,\mathrm{Re}(\gamma_{11})\mathrm{Re}(\gamma_{22}) - \mathrm{Re}(\gamma_{12}\gamma_{21})\}/|\gamma_{12}\gamma_{21}| > 1 \quad (1)$$

and

$$\mathrm{Re}(\gamma_{11}) > 0 \quad \text{or} \quad \mathrm{Re}(\gamma_{22}) > 0, \quad (2)$$

where γ_{ij}'s are twoport immittance parameters [1], [2]. The reason for the necessity of the proviso, however, was left unclear. In terms of S-parameters, the stability criteria equivalent to (1) and (2) are given by

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta_s|^2}{2|S_{12}S_{21}|} > 1, \quad (3)$$

and one of the following auxiliary conditions:

$$|\Delta_s| < 1, \quad (4a)$$

$$B_i = 1 + |S_{ii}|^2 - |S_{jj}|^2 - |\Delta_s|^2 > 0 \quad (i, j = 1, 2 \text{ or } 2, 1), \quad (4b)$$

$$1 - |S_{ii}|^2 > |S_{12}S_{21}| \quad (i = 1 \text{ or } 2), \quad (4c)$$

where $\Delta_s = S_{11}S_{22} - S_{12}S_{21}$ [2–4]. Edwards *et al.* [5], [6] have shown that (3) and (4) are equivalent to a single inequality

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^*\Delta_s| + |S_{12}S_{21}|} > 1. \quad (5)$$

It is to be noted that the criteria (1)–(5) should be satisfied at all frequencies for unconditional stability [7, 8].

While the stability criteria, especially (3) and (4), have been extensively used in microwave circuit design, little reference has been made to Rollett's proviso. Woods [4] showed simple circuit examples with negative resistances whose stability cannot be judged from (3) and (4) due to the violation of the proviso. Recently Platzker *et al.* [7] stimulated attention to the proviso and, by illustrating realistic n-port circuits with multiple active devices, claimed that the role of the stability criteria is quite diminished since the stability of the circuits, i.e. the existence of RHP poles has to be ascertained by other means. There remains, however, a question why the proviso has been seemingly disregarded so far. Thus it is worthwhile to reexamine the necessity and the role of such a proviso from fundamental points of view.

The purpose of this paper is, first to review and redefine Rollett's proviso, and then to introduce a new proviso which includes Rollett's proviso as extreme cases. Implications and roles of the new proviso are discussed.

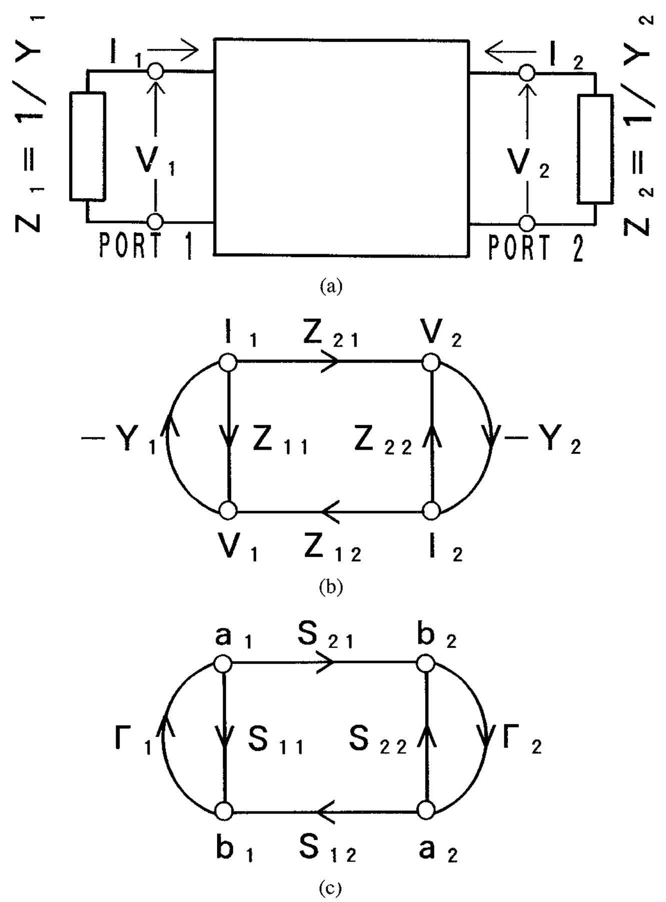

Fig. 1. (a) Linear twoport terminated by impedances $Z_1 = 1/Y_1$, and $Z_2 = 1/Y_2$, (b) signal-flow graph with respect to I_1, I_2, V_1, and V_2, and (c) signal-flow graph with respect to a_1, a_2, b_1, and b_2.

II. REVIEW AND REDEFINITION OF ROLLETT'S PROVISO

Consider a linear twoport terminated by impedances (admittances) $Z_1(Y_1)$ and $Z_2(Y_2)$ as shown in Fig. 1(a). The signal-flow graph with respect to I_1, I_2, V_1 and V_2 is then given by Fig. 1(b), where Z_{ij}'s are the impedance parameters of the twoport. In the case of an open-circuited twoport ($Y_1 = Y_2 = 0$), the necessary and sufficient condition for its stability is that all Z_{ij}'s have no RHP poles as can be seen from Fig. 1(b). Likewise, the necessary and sufficient conditions for the twoport to be stable for $Z_1 = Z_2 = 0$, $Y_1 = Z_2 = 0$, or $Z_1 = Y_2 = 0$ are that all the $Y - (Y_{ij}$'s), $H - (H_{ij}$'s), or G-parameters (G_{ij}'s) have no RHP poles, respectively.

Let us examine the term "characteristic frequency" in Rollett's proviso by taking the open-circuited twoport as an example. Usually the characteristic frequencies are defined as the roots of $\Delta_y = Y_{11}Y_{22} - Y_{12}Y_{21} = 1/(Z_{11}Z_{22} - Z_{12}Z_{21}) = 0$. However, even if they all lie in the LHP, the open-circuited twoport is not always stable, as is evident, for instance, from a case when Z_{21} has RHP poles while $Z_{12} = 0$ [8]. In order to avoid such a confusion, we redefine Rollett's proviso to require that the twoport with open- or short-circuit teminations is stable, or that the immittance parameters have no RHP poles. Then the following theorem ensures that the unconditional stability criteria can be applied if at least one set of Z-, Y-, H- or G-parameters is known to have no RHP poles.

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Theorem 1A: Provided that the immittance parameters γ_{ij}'s of a twoport have no RHP poles, the necessary and sufficient conditions for the twoport to be unconditionally stable are given by

$$\mathrm{Re}\{\gamma_{22}(\omega)\} > 0 \tag{6}$$

and

$$\mathrm{Re}\{\gamma_{\mathrm{in}}(\omega)\} > 0, \tag{7}$$

for all frequencies, where $\gamma_{\mathrm{in}}(\omega)$ is the input impedance $Z_{\mathrm{in}}(\omega)$ when $\gamma_{ij} = Z_{ij}$ or H_{ij} and the input admittance $Y_{\mathrm{in}}(\omega)$ when $\gamma_{ij} = Y_{ij}$ or G_{ij}, respectively, looking into port 1 when port 2 is terminated by arbitrary passive terminations.

Theorem 1B: The conditions (6) and (7) are equivalent to (1) and (2).

Proof: Only the case of $\gamma_{ij} = Z_{ij}$ is proved, since other cases can be proved quite similarly. Let us terminate port 1 and 2 by arbitrary passive admittances $Y_1(= 1/Z_1)$ and $Y_2(=1/Z_2)$, respectively, which have no RHP poles and zeros. Since Z_{ij}'s also have no RHP poles under the postulate, all the branch transmissions are stable. Then the necessary and sufficient condition for the system of Fig. 1(b) to be stable is that the graph determinant of the system

$$\begin{aligned}\Delta &= 1 + Y_1 Z_{11} + Y_2 Z_{22} + Y_1 Y_2 (Z_{11}Z_{22} - Z_{12} Z_{21})\\ &= \Delta_1\Delta_2\end{aligned}$$

has no RHP zeros [9], where

$$\begin{aligned}\Delta_1 &= \Delta(Y_1 = 0) = 1 + Y_2 Z_{22},\\ \Delta_2 &= \Delta/\Delta_1\\ &= 1 + Y_1 Z_{\mathrm{in}},\\ Z_{\mathrm{in}} &= [Z_{11} + Y_2 (Z_{11}Z_{22} - Z_{12}Z_{21})]/(1 + Y_2 Z_{22})\\ &= Z_{11} - Z_{12} Z_{21}/(Z_{22} + Z_2).\end{aligned}$$

For the root of $\Delta = 0$ to remain in the LHP, the roots of $\Delta_1 = 0$ and $\Delta_2 = 0$ must not be in the RHP. Since Y_2Z_{22} has no RHP poles, the Nyquist plot of $Y_2(\omega)Z_{22}(\omega)$ must not encircle the critical point $-1 + j0$ in order for the roots of $\Delta_1 = 0$ not to be in the RHP. This requires $Z_{22}(\omega)$ to lie in the right-half complex plane as $Y_2(\omega)$ can take on any values in the right-half complex plane, which means (6). Otherwise $Y_2(\omega)Z_{22}(\omega)$ can cross the negative real axis on the left of $-1+j0$. Conversely if (6) is satisfied, the roots of $1+Y_2Z_{22} = 0$ remain in the LHP. When (6) is satisfied, Z_{in} has no RHP poles and we can show by similar arguments that (7) is necessary and sufficient for the roots of $1 + Y_1Z_{\mathrm{in}} = 0$ to be in the LHP. This completes the proof of Theorem 1A. Theorem 1B can easily be proved for instance by the method described in [10].

It is worth mentioning that, if $\gamma_{22} = Z_{22}$ has RHP poles for example, (6) or the nonencirclement by the Nyquist plot of Y_2Z_{22} around $-1 + j0$ does not always mean that the zeros of $1 + Y_2Z_{22}$ lie in the LHP, implying that the stability criteria cannot be applied. Rollett's proviso is necessary for avoiding such cases.

III. New Proviso

Let (a_1, b_1) and (a_2, b_2) be the incident and reflected waves at port 1 and 2 in Fig. 1(a), respectively, defined by

$$\begin{aligned}a_i &= (V_i + Z_{0i}I_i)/2\sqrt{Z_{0i}} \qquad (i = 1, 2),\\ b_i &= (V_i - Z_{0i}I_i)/2\sqrt{Z_{0i}} \qquad (i = 1, 2),\end{aligned}$$

where Z_{0i} is the reference impedance at port i. Z_{01} and Z_{02} are assumed here to be positive constant resistances. Using the steady-state S-parameters (S_{ij}'s) that relate a_i's and b_i's, the signal-flow graph for Fig. 1(a) is given by Fig. 1(c), where

$$\Gamma_i = (Z_i - Z_{0i})/(Z_i + Z_{0i}), \quad (i = 1, 2).$$

As in the case of immittance parameters, the necessary and sufficient condition for the system with $\Gamma_1 = \Gamma_2 = 0$ to be stable is that all S_{ij}'s have no RHP poles.

For unconditional stability, the twoport must be stable for arbitrary combinations of passive source ($|\Gamma_1| \leq 1$) and passive load ($|\Gamma_2| \leq 1$). Γ_1 and Γ_2 have no RHP poles. Based upon the following theorem, we propose a new proviso, namely, that the S-parameters defined for at least one pair of positive constant reference impedances have no RHP poles, whence it follows that the twoport is stable for at least one pair of positive constant resistance terminations.

Theorem 2A: Provided that the S-parameters defined for a pair of positive reference impedances have no RHP poles, the necessary and sufficient conditions for a twoport to be unconditionally stable are given by

$$|S_{22}(\omega)| < 1, \tag{8}$$

and

$$|\Gamma_{\mathrm{in}}(\omega)| < 1, \tag{9}$$

for all frequencies, where

$$\Gamma_{\mathrm{in}} = S_{11} + \Gamma_2S_{12}S_{21}/(1 - \Gamma_2S_{22})$$

is the input reflection coefficient looking into port 1 when port 2 is terminated by an arbitrary passive termination.

Theorem 2B: The conditions (8) and (9) are equivalent to (3) and (4).

Proof: When the S-parameters, Γ_1, and Γ_2 have no RHP poles, the necessary and sufficient condition for the system of Fig. 1(c) to be stable is that the graph determinant of the system

$$\begin{aligned}\Delta &= 1 - (\Gamma_1S_{11} + \Gamma_2S_{22} + \Gamma_1\Gamma_2S_{12}S_{21}) + \Gamma_1\Gamma_2S_{11}S_{22}\\ &= \Delta_1\Delta_2\end{aligned}$$

has no RHP zeros [9], where

$$\begin{aligned}\Delta_1 &= \Delta(\Gamma_1 = 0) = 1 - \Gamma_2S_{22},\\ \Delta_2 &= \Delta/\Delta_1\\ &= 1 - \Gamma_1S_{11} - \Gamma_1\Gamma_2S_{12}S_{21}/(1 - \Gamma_2S_{22})\\ &= 1 - \Gamma_1\Gamma_{\mathrm{in}}.\end{aligned}$$

Therefore the twoport is unconditionally stable if and only if Δ has no RHP zeros for arbitrary $|\Gamma_1| \leq 1$ and $|\Gamma_2| \leq 1$, that is, if and only if Δ_1 and Δ_2 have no RHP zeros. For Δ_1 to have no RHP zeros, the Nyquist plot of Γ_2S_{22} should not encircle the critical point $1 + j0$ in the complex plane, which implies

$$|\Gamma_2(\omega)S_{22}(\omega)| < 1. \tag{10}$$

Otherwise a proper choice of the phase of Γ_2 can make Γ_2S_{22} encircle the critical point. For (10) to hold for any $|\Gamma_2| \leq 1$, we have (8). Noting that Δ_2 has no RHP poles when Δ_1 has no RHP zeros, we similarly have

$$|\Gamma_1(\omega)\Gamma_{\mathrm{in}}(\omega)| < 1, \tag{11}$$

for Δ_2 to have no RHP zeros. For (11) to hold for any $|\Gamma_1| \leq 1$, we obtain (9). Conversely we can easily show that (8) and (9) are sufficient for Δ to have no RHP zeros. Thus Theorem 2A has been proved. The proof of Theorem 2B can be found in [5].

If S_{22}, for example, has RHP poles, (10) or the nonencirclement by the Nyquist plot of Γ_2S_{22} around $1 + j0$ does not always mean that the zeros of Δ_1 lie in the LHP. Since Z_{01} and Z_{02} can be chosen arbitrarily between 0 and ∞, we can restate the new proviso as the requirement that the twoport is stable for at least one pair of positive constant resistance terminations. In this sense Rollett's proviso can be considered as the extreme cases of Z_{0i} (i=1,2) $\rightarrow$ 0 or ∞.

IV. Discussion

Let us first consider the general relationship between the proviso and the stability criteria for the following two cases.

A. The Stability Criteria are not Satisfied

The twoport is potentially unstable regardless of the proviso, since the stability criteria must be satisfied if the twoport is unconditionally stable. Needless to say, one cannot tell whether the potential instability is due to the presence of RHP poles in the twoport parameters or not, unless one checks the proviso.

B. The Stability Criteria are Satisfied

The twoport is unconditionally stable if at least any one of the cases in the proviso is found fulfilled. This means that all the other cases in the proviso are satisfied as well. On the contrary, if any one of the cases in the proviso is found unfulfilled, all the other cases are also unfulfilled. In other words, the twoport is unstable for any combinations of $Z_1 = Z_{01}(> 0)$ and $Z_2 = Z_{02}(> 0)$. We can see a simple yet good example for such circumstances in the T-type twoport shown in Fig. 4(b) in [4] that consists of series conductance $G_1(> 0)$, shunt inductance/capacitance/negative conductance $G_3(< 0)$ and series conductance $G_2(> 0)$. One can easily show that all the S- and immittance parameters have RHP poles when $G_1 + G_2 + G_3 < 0$ while the stability criteria (1) and (2) are satisfied.

If the twoport terminated by Z_{01} and Z_{02} does not have any instabilities, the S-parameters have no RHP poles and we can measure the steady-state S-parameters (at least in principle) for all frequencies from 0 to ∞. On the contrary, if there exist some instabilities such as feedback-loop or negative-resistance oscillations, S-parameters cannot be measured consistently at least at the frequencies of the instabilities.

Similarly immittance parameters would have no RHP poles, if we could directly measure them with open- and short-circuit terminations. This, however, is practically impossible at microwave frequencies. Furthermore the steady-state immittance parameters calculated from the measured S-parameters can tell nothing about the locations of their poles, unless we can establish a relevant equivalent circuit of the twoport. This makes it impractical to check Rollett's proviso. The newly stated proviso, however, is readily confirmed by the measurability of the S-parameters.

In microwave amplifier designs, stability checks are mandatory. In the case of an amplifier with a single active device, the first thing to do is to make sure that the S-parameters of the active device have no RHP poles. This is usually ensured by the measurability of the S-parameters as mentioned previously. Then we check whether the active device is unconditionally stable or not using the stability criteria. When the device is found potentially unstable, the next step should be to check the amplifier stability by incorporating the input/output matching circuits into the source and load. This can be performed using the concept of stability circles [11] or the stability analysis method as in [12]. In principle, the application of the stability criteria to the twoport that includes input/output matching circuits might lead to erroneous results, unless the proviso is checked for the twoport.

When we want to check the unconditional stability of a twoport with multiple active devices such as cascaded or parallel-operated amplifiers, we have to first make sure not only that the S-parameters of each active device have no RHP poles, but also that the proviso is met. Confirmation of the proviso is achieved by seeing if the twoport is stable when terminated either by reference impedances, opens or shorts. This can be carried out by the method in [12] when the S-parameters of the active devices have no RHP poles, or also by the method in [7] when the equivalent circuits of the active devices are known.

V. Conclusion

It has been shown that Rollett's proviso should be redefined as that at least one set of immittance parameters have no RHP poles. It can be considered as extreme cases of the new proviso that at least one set of the S-parameters defined for arbitrary positive reference impedances have no RHP poles. The new proviso is ensured if the S-parameters can be measured. Hence the measurability of S-parameters allows us to apply the stability criteria without checking the proviso. In designing twoports with multiple active devices, the stability of the twoports with appropriate terminations has to be ascertained by other means as Platzker et al. suggested [7] before we can confirm the unconditional stability using the stability criteria.

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